Universidad del Desarrollo
Facultad de Educación

Formación basada en Competencias
FORMATO PROGRAMA POR COMPETENCIAS

SYLLABUS OF "TEACHING GEOMETRY AND MEASUREMENT"
Code EBM313

A. GENERAL BACKGROUND

1. Academic Department	Faculty of Education					
2. Career	Pedagogía en Educación Básica mención en Inglés					
3. Code	EBM313					
4. Year / Semester	4th year / 7th semester					
5. Credits	10					
6. Type of subject	Compulsory	x	Elective		Noncompulsory	
7. Term	Bimonthly		Semester	x	Anual	
8. Weekly modules	Theoretical Classes	1	Practical Classes	1	Assistantship	1
9. Academic hours	Classes	68	Assistantships		34	
10. Pre-requisites	None					

B. CONTRIBUTION TO THE GRADUATE PROFILE

This subject seeks that the future teacher of Primary Education develops the essential pedagogical competences to manage and lead a teaching process that promotes different types of geometric thinking. It is expected that, through the implementation of different pedagogical actions, the future teacher orients and guides their primary Education students to explore, manipulate, build, and experiment with the environment, central aspects of spatial sense.

In this course, special attention will be given to the planning of teaching units that contemplate the development of resources and learning goals for the medium term. The future teacher will acquire the necessary tools to simulate and implement sections of said plans, obtaining evidence that will allow you to make decisions about your own professional work.

Likewise, it will promote relationships and connections between topics related to geometry and measurement (essential mathematical actions - such as comparing, ordering, classifying, decomposing, or completing - arise naturally when schoolchildren have to measure magnitudes, for instance).

The subject pays tribute to the generic competences of Ethics, Analytical Vision and Efficiency, and to the following competences of the graduate profile:

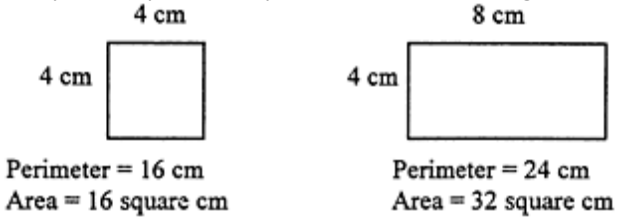
- Competence 1. Engages all students with their own learning, through purposeful and challenging learning experiences, in both English and Spanish, demonstrating high disciplinary mastery.
- Competence 2. Systematically evaluates, analyzes and communicates the progress of students based on evidence, and uses data to improve the teaching-learning process.
- Competence 5. Demonstrates professionalism in teaching, for the benefit of student learning.

It is part of the curricular axis of Disciplinary Pedagogical Training of the Mathematics line. The subject is located in the Bachelor's degree cycle.

C. PURPOSE OF THE COURSE

Read the following situation[1]:

Imagine that one of your students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed Figure 1 increases, the area also increases. She shows you this picture to prove what she is doing:



- How would you respond to this student?

Students bring up novel ideas and claims in their mathematics classes. Sometimes teachers know whether a student's claim is valid, but sometimes they do not. The perimeter and area of a figure are two different measures. The perimeter is a measure of the length of the boundary of a figure (in the case of a rectangle, the sum of the lengths of the sides of the figure), while the area is a measure of the size of the figure. Because the calculations of both measures are related to the sides of a figure, the student claimed that they were correlated.

The above situation highlights one of the great challenges of teaching geometry and measurement: going beyond calculations or procedures. In this course, we seek to develop: (a) a spatial sense and geometric reasoning that allows us to understand how students think and reason about form and space; and (b) a deep understanding of the curricular framework that governs the learning of geometry and measurement for students from 6-12 years old.

[1] Ma, L. (2010). *Knowing and teaching elementary mathematics. Teachers' Understanding of Fundamental Mathematics in China and the United States* (p. 72).

D. COMPETENCE AND GENERAL LEARNING RESULTS DEVELOPED BY THE SUBJECT

Graduate profile competences	**Graduate profile Sub-competences**	**Course learning outcome**
C1. Engages all students with their own learning, through purposeful and challenging learning experiences, in both English and Spanish, demonstrating high disciplinary mastery.	1.1. Applies a solid and updated base of disciplinary and didactic knowledge to design and execute learning experiences.	1.1. Demonstrates the mathematical knowledge for teaching involved in the teaching of geometry and measurement, in order to demonstrate a high domain of disciplinary and pedagogical knowledge.
	1.2. It implements the processes that allow the acquisition of all basic education learning, both in Spanish and in English, showing a high command of the English language.	1.2. Uses different types of mathematical representations (manipulatives, pictures, symbolic, digitals) considering: its relationship with the comprehension of concepts and procedures involved in geometry and measurement; and the diversity of students.
	1.3. Plan learning experiences aligned with the current curriculum, sequenced, meaningful, challenging and approachable, considering how each subject is taught, the evidence of research and professional experience.	1.3. Critically analyzes the progression of the contents referred to geometry and measurement in the mathematical curriculum of primary Education.
	1.3. Plan learning experiences aligned with the current curriculum, sequenced, meaningful, challenging and approachable, considering how each subject is taught, the evidence of research and professional experience. --- 1.5. Develops higher thinking skills in all students, such as critical, creative, and metacognitive thinking.	1.4. Designs mathematical tasks for the promotion and development of geometric thinking and measurement, as well as to develop skills of argumentation and communication, in order to provide meanings and connect mathematical ideas.
	1.3. Plan learning experiences aligned with the current curriculum, sequenced, meaningful, challenging and approachable, considering how each subject is taught, the evidence of research and professional experience. --- 1.6. Select and use teaching resources and ICTs, aligned with the learning objectives of each subject and the evidence of educational research and reflection of their own practices	1.5. Plans learning goals for a class, including the necessary resources that contribute to these goals, in order to promote geometric thinking.
	1.3. Plan learning experiences aligned with the current curriculum, sequenced, meaningful, challenging and approachable, considering how each subject is taught, the evidence of research and professional experience.	1.6. Creates a planning by didactic units, demonstrating coherence, significance and relevance for mathematical learning and its diversity.

	1.4. It stimulates the learning of all, communicating effectively, using different patterns of interaction and teaching models in the classroom.	1.7. Explains and models mathematical ideas and procedures when teaches, considering the whole-group students as well as small-group students, ensuring access to equitable learning for all students.
	1.6. Select and use teaching resources and ICTs, aligned with the learning objectives of each subject and the evidence of educational research and reflection of their own practices	1.8. Critically analyzes current technological resources (software, for instance) that promote the development of geometric thinking and measurement, responding to the immediate needs of the environment.
	1..7. It carries out actions that address special needs and talents, responding to the diversity of students and developing their maximum potential.	1.9. Understands the different difficulties or errors in learning geometry and measurement and propose different strategies to address them.
C2. Systematically evaluates, analyzes and communicates the progress of students based on evidence, and uses the data to improve the teaching-learning process.	2.1. Continuously evaluate and record student learning, building and/or using the most pertinent methods and instruments for it.	2.1. Designs deliberate questions to assess and improve their students' geometric reasoning, as well as to make sense of important mathematical ideas and relationships (Sub competence 2.1).
	2.1. Continuously evaluate and record student learning, building and/or using the most pertinent methods and instruments for it.	2.2. Designs activities and different instruments to evaluate the ability to solve problems related to topics linked to the teaching of geometry and measurement.
	2.2. Improve student learning by continuously analyzing assessment data.	2.3. Adapts teaching based on evidence of their students' thinking, justifying the decisions made.
C5. Demonstrates professionalism in their teaching, for the benefit of student learning.	5.3. Acts with integrity and responsibility when approaching all professional tasks.	5.1. Prepares its personal or collaborative works with stringency, demonstrating quality, neatness, order and academic honesty.

E. COMPETENCE UNITS

Unit 1: Disciplinary knowledge of geometry and measurement

Observe[2] the following Figures:

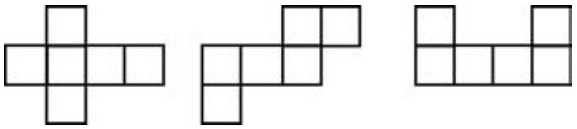


Figure A

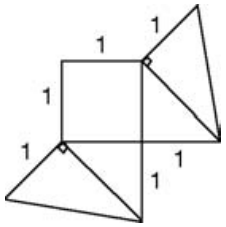


Figure B

- Which of the nets in Figure A can fold to make a cube?
- How many different nets can you find for a cube?
- What shape do you get when you fold up the net in Figure B?

The principal value in such a task is to develop powers of visualisation, concentration, discernment and mental agility. Did you take time to allow yourself to manipulate the nets mentally to see if they would make a cube? Notice the difference in effect of the two types of task: 'which of these has … ?' and 'in how many ways can you … ?'. In the first you seed an idea, while in the other you set a challenge. Learners constructing their own nets can challenge each other as to whether a diagram really is a net. Imagining different nets for surfaces you encounter, such as prisms (including cylinders) and pyramids, exercises your powers to imagine creatively. Note that a net for a cylinder is forced to have a disc attached 'at a point' to a rectangle.

This course addresses the essential elements to guide learning related to geometry and measurement from 1st to 6th grade of primary school. Specifically, in this unit the key conceptual and procedural concepts that you must handle to address teaching and learning sequences are worked on.

Course learning outcome	Evidence of competence / Performance Criteria
1.1.1. Demonstrates the mathematical knowledge for teaching involved in the teaching of geometry and measurement, in order to demonstrate a high domain of disciplinary and pedagogical knowledge.	**1.1.1.1**. Define precisely the concepts associated with intuitive geometry and deductive geometry, elementary geometry of the plane, geometry of space, and geometric movements in the plane. **1.1.1.2**. Applies in a pertinent and justified way properties of geometric bodies and plane figures. **1.1.1.3.** Model everyday situations using spatial relationships and views of 3D and 2D figures. **1.1.1.4.** Manage, draw and build geometric elements

[2] Johnston-Wilder, S. y Mason, J. (2005). *Developing hinking in geometry* (p. 98). Sage.

1.1.2. Uses different types of mathematical representations (manipulatives, pictures, symbolic, digitals) considering: its relationship to understanding the concepts and procedures involved in geometry and measurement; and the diversity of students.	**1.1.2.1.** Give precise meaning and connect multiple representations of objects and figures in 3D and 2D. **1.1.2.2.** Develop mathematical arguments about geometric relationships based on different representations.

Conceptual content.

- The location of an object in the space.
- Distances, displacements, angles and turns as reference elements.
- Cartesian coordinate systems.
- Relations between geometric elements: parallelism, perpendicularity, intersection of straight lines.
- Plane shapes: figures and their elements (polygons and circumference)
- Relations between the elements of a figure and the figures among themselves.
- Spatial shapes. Geometric bodies and their elements: vertices, edges and faces.
- Cube, sphere, prisms, pyramids, cones and cylinders.
- Relations between geometric bodies.
- Regularities and symmetries in flat and spatial forms.
- The elementary representation of space: plans, maps, models, scales.
- Drawing instruments.

Procedures content.

- Description of the situation and position of an object in space in relation to oneself and/or other appropriate reference points.
- Representation and reading of points in Cartesian coordinate systems.
- Preparation, interpretation and verbal description of sketches and itineraries.
- Reading, interpretation and description of maps.
- Use of the usual drawing instruments for the construction and exploration of geometric shapes.
- Appropriate use of basic geometric vocabulary in the description of familiar objects.

Attitudinal content.

- Assessment of the use of reference systems and spatial representation in daily activities.
- Sensitivity and taste for the elaboration and for the careful presentation of geometric constructions.
- Precision and care in the use of drawing instruments and a favorable disposition to search for alternative instruments.

Recursos de aprendizaje obligatorios:

Bibliografía básica:

- Godino, J. D. (2004). Didáctica de las matemáticas para maestros. España: Departamento de Didáctica de las Matemáticas, University of Granada, Spain. Open acces: https://www.ugr.es/~jgodino/edumat-maestros/manual/9_didactica_maestros.pdf
- Gravemeijer, K., Figueiredo, N., Feijs, E., Van Galen, F., Keijzer, R. y Munk, F. (2016). *Measurement and Geometry in Upper Primary School*. Países Bajos: Sense Publishers.
- Reyes, C., Disset, L. y Gormaz, R. (2013). *REFIP Matemática: Geometría para futuros profesores de Educación Básica*. Santiago: Ediciones SM. Open access: http://www.smconecta.cl/refip/

Bibliografía complementaria:

- Chick, L., Holmesn, A. S., McClymonds, N., Musick, S., Reynolds, P. y Schultz, G. (2008). Geometry and measurement. *Teaching Children Mathematics*, 14(7), 408-409.
- Hede, J. T., & Bostic, J. D. (2014). Connecting the Threads of Area and Perimeter. *Teaching Children*

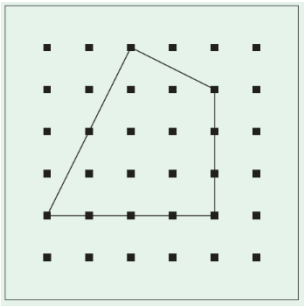
Mathematics, *20*(7), 418-425. https://doi.org/10.5951/teacchilmath.20.7.0418

- Larraín, M. & Chandía. E. (2012). *La enseñanza de la geoemetría en la formación inicial de profesores de educación básica: una propuesta metodológica*. Santiago: Universidad del Desarrollo.
- Ministerio de Educación (2012). Bases Curriculares para la Educación Básica. Santiago, Chile: Unidad de Currículum y Evaluación.
- Porkess, R. (Editor) (2014). Geometry and measures. Reino Unido: Hodder Education.
- Schettino, C. (2011). Teaching Geometry through Problem-Based Learning. The Mathematics Teacher, 105(5), 346-351. http://www.jstor.org/stable/10.5951/mathteacher.105.5.0346 .

Informáticos:
https://www.nctm.org/sem/
https://www.nctm.org/Classroom-Resources/More-Online-Resources-from-NCTM/
https://www.nctm.org/pows/
https://www.nctm.org/crcc/
https://www.nctm.org/ARCs/
https://www.nctm.org/Standards-and-Positions/Focus-in-High-School-Mathematics/Reasoning-and-Sense-Making-Task-Library/

Softwares
https://www.didax.com/math/virtual-manipulatives.html
https://mathigon.org/polypad
https://phet.colorado.edu/en/simulations/filter?subjects=math&type=html&sort=alpha&view=grid
https://www2.ual.es/neotrie/

UNIT 2: Teaching and learning geometry



Suppose[3] that your elementary school students are working in pairs and they have to work with the geoboard. Specifically, one student constructs a shape on a geoboard out of the sight of his partner. This student (the described) will describe the shape to his partner using only words, not hand gestures. The partner will attempt to replicate the shape on her own geoboard based on the description, but do it out of the sight of the describer. Repeat this activity with the partners changing roles.

- What could be the key elements that allow a successful description of the figure?

In the late 1950s in the Netherlands, two mathematics teachers, Pierre van Hiele and Dieke van Hiele-Geldof, husband and wife, put forth a theory of development in geometry based on their own teaching and research. They observed that in learning geometry, students seem to progress through a sequence of five reasoning levels, from holistic thinking to analytical thinking to rigorous abstract mathematical deduction. The van Hieles described the five levels of reasoning which are part of this unit.

In this unit, we are going to delve into the different theoretical contributions that have deepened the way students' reason geometrically. Specifically, we will design different actions that allow students to find relationships that favor the development of thinking based on increasingly sophisticated relationships and demonstrations.

Course learning outcome	Evidence of competence / Performance Criteria
1.1. Demonstrates the mathematical knowledge for teaching involved in the teaching of geometry and measurement, in order to demonstrate a high domain of disciplinary and pedagogical knowledge.	1.1.1. Recognizes the main theories related to teaching and learning geometry 1.1.2. Identifies the principles elements of Van Hiele's geometric reasoning model and visual skills for geometric learning
1.2. Uses different types of mathematical representations (manipulatives, pictures, symbolic, digitals) considering: its relationship to understanding the concepts and procedures involved in geometry and measurement; and the diversity of students.	1.2.1. Recognizes concrete representations that favor the learning of geometry 1.2.2. Distinguish the role of different representations to explore the different properties of 2D and 3D shapes. 1.2.3. Mathematically demonstrates geometric properties through manipulative, pictorial, and symbolic representations. 1.2.4. It communicates the potentialities and deficiencies of the different types of representations, according to the mathematical content to which they are taxed.
1.3. Critically analyzes the progression of the contents related to geometry and measurement in the mathematical curriculum of primary Education.	1.3.1. Identifies the contents related to geometry in the study programs from first to sixth grade.

[3] Musser, G. L., Burger, W. F., and Petereson.E. (2011). *Mathematics For Elementary Teachers: A contemporary approach* (p. 577): Wiley,

	1.3.1. Organizes the procedural treatment of geometry in the study programs from first to sixth grade. 1.3.1. Analyzes the treatment of mathematical skills, referred to geometry in the study programs from first to sixth grade
1.4. Designs mathematical tasks for the promotion and development of geometric thinking and measurement, as well as to develop skills of argumentation and communication, in order to provide meanings and connect mathematical ideas.	1.4.1. Planning a problem situation from 1st to 6th grade that promote geometric thinking. 1.4.2. Organize a sequence of mathematical tasks that favors the development of visualization skills and competencies in its students.
1.5. Plans learning goals for a class, including the necessary resources that contribute to these goals, in order to promote geometric thinking.	1.5.1. Demonstrates understanding of the main conceptual, procedural and attitudinal contents involved in an OA from Geometry topic. 1.5.2. Design specific class objectives that account for a progression in the development of geometric thinking. 1.5.3. Assesses student learning with an emphasis on decision making.
1.6. Creates a planning by didactic units, demonstrating coherence, significance and relevance for mathematical learning and its diversity.	1.6.1 Plan learning sequences. 1.6.2. Design or select learning activities that contribute to the selected objective.
1.7. Explains and models mathematical ideas and procedures when teaches, considering the whole-group students as well as small-group students, ensuring access to equitable learning for all students.	1.7.1. Communicates geometric concepts and procedures accurately. 1.7.2. Use different representations to support his explanations. 1.7.3. Select examples that favor different types of geometric reasoning
1.8. Critically analyzes current technological resources (software, for instance) that promote the development of geometric thinking and measurement, responding to the immediate needs of the environment.	1.8.1. Select and adapt different technological tools to develop and promote geometric thinking skills. 1.8.2. Design learning activities base don ICTs that promote the four mathematical skills proposed by the national curriculum: represent, solve problems, argue and communicate, and model.
1.9. Interpret the different difficulties or errors in learning geometry and measurement and propose different strategies to address them.	1.9.1. Interpret the origin of different errors and/or difficulties in the teaching and learning of Euclidean geometry. 1.9.2. Design strategies to address errors and/or difficulties associated with the teaching of Euclidean geometry, considering the diversity of students.
2.1. Designs deliberate questions to assess and improve their students' geometric reasoning, as well as to make sense of important mathematical ideas and relationships.	2.1.1. Organize the different types of reasoning that can emerge from students and identify ways to collect evidence on said reasoning. 2.1.2. Use meaningful questions that encourage reflection and visualization 2.1.3. Makes decisions based on the evidence obtained from the work of their students.
2.3. Adapts teaching based on evidence of their students' thinking, justifying the decisions made.	2.3.1. Make modifications to teaching resources based on evidence. 2.3.2. It justifies its decisions for change based on the literature on Euclidean geometry, the curricular framework and current research.

5.1. Prepares its personal or collaborative works with stringency, demonstrating quality, neatness, order and academic honesty.	5.1.1. Prepares clear, orderly teaching material with a high disciplinary level. 5.1.2. Consider the opinion of others to enrich your teaching process

Conceptual content.

- The development of geometric thinking (spacial sense and geometric thinking / The van Hiele levels of geometric thought).
  - Implications for Instruction of Van Hiele's theory.
  - Shapes and Properties for Level-0 Thinkers.
  - Shapes and Properties for Level-1 Thinkers.
  - Shapes and Properties for Level-2 Thinkers.
  - Transformations for Level-0 Thinkers.
  - Transformations for Level-1 Thinkers.
  - Transformations for Level-2 Thinkers.
  - Location for Level-1 Thinkers.
  - Location for Level-2 Thinkers.
  - Visualization for Level-0 Thinkers.
  - Visualization for Level-1 Thinkers.
  - Visualization for Level-2 Thinkers.
- Learning about shapes and properties; transformations; location; visualization.
- Difficulties and errors in learning geometry
- Materials, resources and tasks for the work of geometry
- Curricular progression.
- Technological tools that promote visualization (e.g. Geogebra).
- Spatial Sense and Geometric Reasoning.

Procedure content.

- Characterize and define geometric elements to design learning sequences
- Manage, draw and build geometric elements to support their teaching explanations.
- Classify figures and geometric bodies and state their properties using different types of representations.
- Develop mathematical arguments about geometric relationships
- Obtain empirical and formal measurements of bodies and figures, and relate them to each other
- Justify and prove some properties of bodies and figures
- Pose and solve problems using geometric elements
- Find relationships between geometric concepts
- Know and handle materials and resources for learning geometry

Attitudinal content.

- Curiosity to investigate and explore the meanings of geometric representations.
- Interest in the discovery and formulation of properties of Euclidean geometry.
- Development of the predisposition to justify geometric properties.
- Critical attitude towards the possibility of admitting "a priori" that a formulated relationship is true.
- Sensitivity and interest in the messages of geometric nature, appreciating the usefulness of operations in everyday life.

Recursos de aprendizaje obligatorios:

Bibliografía básica:

- Godino, J. D. (2004). Didáctica de las matemáticas para maestros. España: Departamento de Didáctica de las Matemáticas, University of Granada, Spain. Open acces: https://www.ugr.es/~jgodino/edumat-

maestros/manual/9_didactica_maestros.pdf

- Gravemeijer, K., Figueiredo, N., Feijs, E., Van Galen, F., Keijzer, R. y Munk, F. (2016). *Measurement and Geometry in Upper Primary School*. Países Bajos: Sense Publishers.
- Reyes, C., Disset, L. y Gormaz, R. (2013). *REFIP Matemática: Geometría para futuros profesores de Educación Básica*. Santiago: Ediciones SM. Open access: http://www.smconecta.cl/refip/
- Sack, J. y Vazquez, I. (2016). A 3D visualization teaching-learning trajectory for elementary grades children. Países Bajos: Springer. doi: 10.1007/978-3-319-29799-6
- Jurdak, M. (2016). *Learning and teaching real world problem solving in school mathematics*. The Netherland: Springer.
- Clements, D. H., Swaminathan, S., Zeitler, M. A. y Sarama, J. (1999). Young Children's Concepts of Shape. *Journal for Research in Mathematics Education*, *30*(2), 199-212.
- Howse, M. E. y Howse, T. D. (2015). Linking the Van Hiele theory to instruction. *Teaching Children Mathematics*, *21*(5), 304-313.

Bibliografía complementaria:

- Aldon, G. & Trgalova, J. (Eds.). (2019). *Teachnology in mathematics teaching*. The Netherlands: Springer.
- Andreasen, J. B. & Haciomeroglu, E. S. (2014). Engaging geometry students through technology. *Mathematics Teaching in the middle school*, *19*(5), 308-310.
- Chapin, S., O´Connor, & Anderson, N.C. (2005). Classroom discussions: Using math talk in elementary classrooms. Recuperado en http://mail.mathsolutions.com/documents/978- 1-935099-01-7_L.pdf
- Chick, L., Holmesn, A. S., McClymonds, N., Musick, S., Reynolds, P. y Schultz, G. (2008). Geometry and measurement. *Teaching Children Mathematics*, 14(7), 408-409.
- Gonzato, M., Godino, J. D. y Neto, T. (2011). Evaluación de conocimientos didáctico-matemáticos sobre la visualización de objetos tridimensionales. *Educación Matemática*, *23*, (3), 5-37.
- Hede, J. T., & Bostic, J. D. (2014). Connecting the Threads of Area and Perimeter. *Teaching Children Mathematics*, *20*(7), 418-425. https://doi.org/10.5951/teacchilmath.20.7.0418
- Herbst, P., Cheah, U. H., Richard, P. R., & Jones, K. (Eds.). (2018). *International perspectives on the teaching and learning of geometry in secondary school*. Países Bajos: Springer.
- International association for the evaluation of educational achievement (2009). Released ítems. Mathmatics – Forth Grade. Chestnut Hill, MA: IAE. Recuperado en: http://timssandpirls.bc.edu/timss2007/items.html
- Larraín, M. & Chandía. E. (2012). *La enseñanza de la geoemetría en la formación inicial de profesores de educación básica: una propuesta metodológica*. Santiago: Universidad del Desarrollo.
- Ministerio de Educación (2012). Bases Curriculares para la Educación Básica. Santiago, Chile: Unidad de Currículum y Evaluación.
- National Council of Teachers of Mathematics. (2011). Teaching geometry. *The Mathematics Teacher*, *105*(4), 244.
- Oldknow, A. & Knights, C. (2011). *Mathematics Education with digital Technology.* New York, NY: Continuum.
- Porkess, R. (Editor) (2014). Geometry and measures. Reino Unido: Hodder Education.
- Schettino, C. (2011). Teaching Geometry through Problem-Based Learning. The Mathematics Teacher, 105(5), 346-351. http://www.jstor.org/stable/10.5951/mathteacher.105.5.0346 .
- Usiskin, Z. (2010). *Future Curricular Trends in School Algebra And Geometry: Proceedings of A Conference*. Charlotte, NC: Information Age Publishing.

Informáticos:

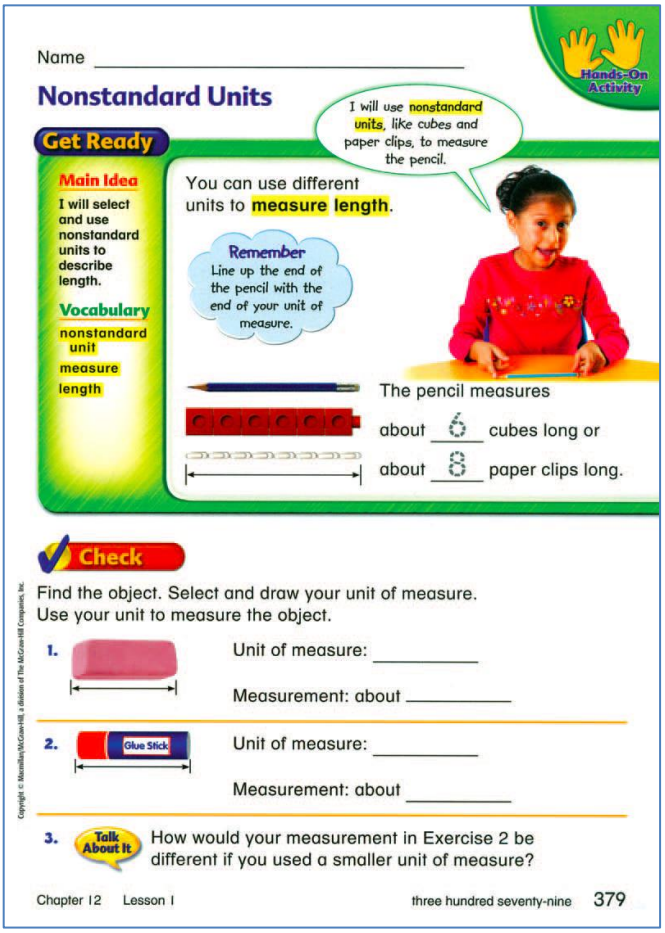
https://www.nctm.org/sem/
https://www.nctm.org/Classroom-Resources/More-Online-Resources-from-NCTM/
https://www.nctm.org/pows/
https://www.nctm.org/crcc/

https://www.nctm.org/ARCs/
https://www.nctm.org/Standards-and-Positions/Focus-in-High-School-Mathematics/Reasoning-and-Sense-Making-Task-Library/

Softwares
https://www.didax.com/math/virtual-manipulatives.html
https://mathigon.org/polypad
https://phet.colorado.edu/en/simulations/filter?subjects=math&type=html&sort=alpha&view=grid
https://www2.ual.es/neotrie/

UNIT 3: Teaching and learning of magnitudes and their measurement.

Let's look at the next page of a math textbook:



Name _____

Nonstandard Units

Hands-On Activity

I will use nonstandard units, like cubes and paper clips, to measure the pencil.

Get Ready

Main Idea
I will select and use nonstandard units to describe length.

Vocabulary
nonstandard unit
measure
length

You can use different units to **measure length**.

Remember
Line up the end of the pencil with the end of your unit of measure.

The pencil measures
about 6 cubes long or
about 8 paper clips long.

Check

Find the object. Select and draw your unit of measure.
Use your unit to measure the object.

1. Unit of measure: _____
Measurement: about _____

2. Glue Stick
Unit of measure: _____
Measurement: about _____

3. Talk About It
How would your measurement in Exercise 2 be different if you used a smaller unit of measure?

Chapter 12 Lesson 1 three hundred seventy-nine 379

Measuring is comparing. This is a dynamic process with great challenges, given that many times the focus is on the procedural rather than on the concepts involved. The page 379 (See Figure) shows one of the first approaches that the students of the first courses have when approaching the measurement process: the use of non-standardized measures. This process is gradual and requires making sense of the process of determining the magnitude of a measurement. Now, how do we make this process meaningful and go beyond determining a number?

In this unit, we are going to delve into the idea of measure reasoning. To do this, we will identify the main contributions of the didactics of mathematics to understand how this process develops in students, based on curricular, theoretical and pedagogical frameworks.

Course learning outcome	Evidence of competence / Performance Criteria

1.1. Demonstrates the mathematical knowledge for teaching involved in the teaching of geometry and measurement, in order to demonstrate a high domain of disciplinary and pedagogical knowledge.	1.1.1. Recognizes the main theories related to teaching and learning measurement of quantities 1.1.2. Identifies the principles elements of Van Hiele's geometric reasoning model and visual skills for geometric learning
1.2. Uses different types of mathematical representations (manipulatives, pictures, symbolic, digitals) considering: its relationship to understanding the concepts and procedures involved in geometry and measurement; and the diversity of students.	1.2.1. Recognizes concrete representations that favor the learning of measurement of quantities 1.2.2. Distinguish the role of different representations to explore ways to determinate the measurement of quantities, in the context of 2D y 3D figures. 1.2.3. Demonstrates geometric properties through manipulative, pictorial, and symbolic representations. 1.2.4. Communicate the potentialities and deficiencies of the different types of representations, according to the geometric content to which they are taxed.
1.4. Designs mathematical tasks for the promotion and development of geometric thinking and measurement, as well as to develop skills of argumentation and communication, in order to provide meanings and connect mathematical ideas.	1.4.1. Planning a problem situation from 1st to 6th grade that promote a significative process of measure quantities 1.4.2. Organize a sequence of mathematical tasks that favors the development of measurement of quantities and competencies in its students.
1.5. Plans learning goals for a class, including the necessary resources that contribute to these goals, in order to promote geometric thinking.	1.5.1. Demonstrates understanding of the main conceptual, procedural and attitudinal contents involved in an OA from Measurement topic. 1.5.2. Design specific class objectives that account for a progression in the development of measurement thinking. 1.5.3. Assesses student learning with an emphasis on decision making.
1.6. Creates a planning by didactic units, demonstrating coherence, significance and relevance for mathematical learning and its diversity.	1.6.1 Plan learning sequences. 1.6.2. Design or select learning activities that contribute to the selected objective.
1.7. Explains and models mathematical ideas and procedures when teaches, considering the whole-group students as well as small-group students, ensuring access to equitable learning for all students.	1.7.1. Communicates measurement concepts and procedures accurately. 1.7.2. Use different representations to support his explanations. 1.7.3. Select examples that favor different types of measurement reasoning.
1.8. Critically analyzes current technological resources (software, for instance) that promote the development of geometric thinking and measurement, responding to the immediate needs of the environment.	1.8.1. Select and adapt different technological tools to develop and promote measurement thinking skills. 1.8.2. Design learning activities base don ICTs that promote the four mathematical skills proposed by the national curriculum: represent, solve problems, argue and communicate, and model.
1.9. Interpret the different difficulties or errors in learning geometry and measurement and propose different strategies to address them.	1.9.1. Interpret the origin of different errors and/or difficulties in the teaching and learning of measurement of quantities. 1.9.2. Design strategies to address errors and/or difficulties associated with the teaching of measurement of quantities, considering the diversity of students.
2.2. Designs activities and different instruments to evaluate the ability to solve problems related to	2.2.1. Identify different ways to formatively assess their students' learning of geometry and measurement.

topics linked to the teaching of geometry and measurement	2.2.2. Adapt different resources that allow collecting evidence on the learning of their students (study texts, for example).
2.3. Adapts teaching based on evidence of their students' thinking, justifying the decisions made.	2.3.1. Make modifications to teaching resources based on evidence. 2.3.2. It justifies its decisions for change based on the literature on Euclidean geometry, the curricular framework and current research.
5.1. Prepares its personal or collaborative works with stringency, demonstrating quality, neatness, order, and academic honesty.	5.1.1. Prepares clear, orderly teaching material with a high disciplinary level. 5.1.2. Consider the opinion of others to enrich your teaching process

Conceptual content.

- • The meaning and process of measurement
- • Teaching and learning of the measurement of quantities
- • Calculation and estimation
- • Difficulties and errors in learning magnitudes and their measurement
- • Design of mathematical tasks.
- • Asking deliberate questions

Procedure content.

- Use of geometric instrument to determine the measurement of geometric figures
- Manage, draw and build geometric elements to support their teaching explanations.
- Classify figures and geometric bodies and state their properties using different types of representations.
- Develop mathematical arguments about the process of measurement.
- Obtain empirical and formal measurements of bodies and figures, and relate them to each other
- Justify and prove some properties of bodies and figures
- Pose and solve problems using geometric elements
- Know and handle materials and resources for learning measurement.

Contenidos actitudinales.

- Curiosity to investigate and explore regularities in measurement of geometric content.
- Development of the predisposition to justify geometric properties in the context of measurement quantities.
- Critical attitude towards the possibility of admitting "a priori" that a formulated relationship is true.
- Sensitivity and interest in the messages of geometric nature, appreciating the usefulness of operations in everyday life.

Recursos de aprendizaje obligatorios:

Bibliografía básica:

- Godino, J. D. (2004). Didáctica de las matemáticas para maestros. España: Departamento de Didáctica de las Matemáticas, University of Granada, Spain. Open acces: https://www.ugr.es/~jgodino/edumat-maestros/manual/9_didactica_maestros.pdf
- Gravemeijer, K., Figueiredo, N., Feijs, E., Van Galen, F., Keijzer, R. y Munk, F. (2016). *Measurement and Geometry in Upper Primary School*. Países Bajos: Sense Publishers.
- Reyes, C., Disset, L. y Gormaz, R. (2013). *REFIP Matemática: Geometría para futuros profesores de Educación Básica*. Santiago: Ediciones SM. Open access: http://www.smconecta.cl/refip/
- Sack, J. y Vazquez, I. (2016). A 3D visualization teaching-learning trajectory for elementary grades children. Países Bajos: Springer. doi: 10.1007/978-3-319-29799-6
- Jurdak, M. (2016). *Learning and teaching real world problem solving in school mathematics*. The Netherland:

Springer.

- Clements, D. H., Swaminathan, S., Zeitler, M. A. y Sarama, J. (1999). Young Children's Concepts of Shape. *Journal for Research in Mathematics Education*, *30*(2), 199-212.
- Howse, M. E. y Howse, T. D. (2015). Linking the Van Hiele theory to instruction. *Teaching Children Mathematics*, *21*(5), 304-313.

Bibliografía complementaria:

- Aldon, G. & Trgalova, J. (Eds.). (2019). *Teachnology in mathematics teaching*. The Netherlands: Springer.
- Andreasen, J. B. & Haciomeroglu, E. S. (2014). Engaging geometry students through technology. *Mathematics Teaching in the middle school*, *19*(5), 308-310.
- Chapin, S., O´Connor, & Anderson, N.C. (2005). Classroom discussions: Using math talk in elementary classrooms. Recuperado en http://mail.mathsolutions.com/documents/978- 1-935099-01-7_L.pdf
- Chick, L., Holmesn, A. S., McClymonds, N., Musick, S., Reynolds, P. y Schultz, G. (2008). Geometry and measurement. *Teaching Children Mathematics*, 14(7), 408-409.
- Gonzato, M., Godino, J. D. y Neto, T. (2011). Evaluación de conocimientos didáctico-matemáticos sobre la visualización de objetos tridimensionales. *Educación Matemática*, *23*, (3), 5-37.
- Hede, J. T., & Bostic, J. D. (2014). Connecting the Threads of Area and Perimeter. *Teaching Children Mathematics*, *20*(7), 418-425. https://doi.org/10.5951/teacchilmath.20.7.0418
- Herbst, P., Cheah, U. H., Richard, P. R., & Jones, K. (Eds.). (2018). *International perspectives on the teaching and learning of geometry in secondary school*. Países Bajos: Springer.
- International association for the evaluation of educational achievement (2009). Released ítems. Mathmatics – Forth Grade. Chestnut Hill, MA: IAE. Recuperado en: http://timssandpirls.bc.edu/timss2007/items.html
- Larraín, M. & Chandía. E. (2012). *La enseñanza de la geoemetría en la formación inicial de profesores de educación básica: una propuesta metodológica*. Santiago: Universidad del Desarrollo.
- Ministerio de Educación (2012). Bases Curriculares para la Educación Básica. Santiago, Chile: Unidad de Currículum y Evaluación.
- National Council of Teachers of Mathematics. (2011). Teaching geometry. *The Mathematics Teacher*, *105*(4), 244.
- Oldknow, A. & Knights, C. (2011). *Mathematics Education with digital Technology.* New York, NY: Continuum.
- Porkess, R. (Editor) (2014). Geometry and measures. Reino Unido: Hodder Education.
- Schettino, C. (2011). Teaching Geometry through Problem-Based Learning. The Mathematics Teacher, 105(5), 346-351. http://www.jstor.org/stable/10.5951/mathteacher.105.5.0346 .
- Usiskin, Z. (2010). *Future Curricular Trends in School Algebra And Geometry: Proceedings of A Conference*. Charlotte, NC: Information Age Publishing.

Informáticos:

https://www.nctm.org/sem/
https://www.nctm.org/Classroom-Resources/More-Online-Resources-from-NCTM/
https://www.nctm.org/pows/
https://www.nctm.org/crcc/
https://www.nctm.org/ARCs/
https://www.nctm.org/Standards-and-Positions/Focus-in-High-School-Mathematics/Reasoning-and-Sense-Making-Task-Library/

Softwares

https://www.didax.com/math/virtual-manipulatives.html

https://mathigon.org/polypad
https://phet.colorado.edu/en/simulations/filter?subjects=math&type=html&sort=alpha&view=grid
https://www2.ual.es/neotrie/

F. TEACHING STRATEGIES

This course seeks to expose, articulate and model different methodological strategies that highlight the active role of teachers in training with their learning. We consider different methodological strategies that favor a mathematical discussion, promoting a critical interaction with the topics of the subject. Therefore, the teaching methodology has a strong emphasis on practice, so the development of practical work (individual or in group) and active participation become essential elements. Specifically, the structure course based on various methodologies, which include:

1) Group work and debates among the same students.
2) Work guides developed in classes.
3) Simulated practices that promote decision-making.
4) Modeling of certain mathematical contents through the use of manipulative materials.
5) Analysis activities and cases critiques (real and supposed) during most of the classes.
6) Analysis and contrast of disciplinary and didactic information sources.
7) Conferences supported with technologies such as Power Point, Prezi, Geogebra, among others.

G. ASSESSMENT STRATEGIES

The learning assessment is organized into:

- Tests.
- Disciplinary workshops.
- Planning of teaching units.
- Final exam.

H. RESOURCES

Bibliografía básica:

- Godino, J. D. (2004). Didáctica de las matemáticas para maestros. España: Departamento de Didáctica de las Matemáticas, University of Granada, Spain. Open acces: https://www.ugr.es/~jgodino/edumat-maestros/manual/9_didactica_maestros.pdf
- Gravemeijer, K., Figueiredo, N., Feijs, E., Van Galen, F., Keijzer, R. y Munk, F. (2016). *Measurement and Geometry in Upper Primary School*. Países Bajos: Sense Publishers.
- Reyes, C., Disset, L. y Gormaz, R. (2013). *REFIP Matemática: Geometría para futuros profesores de Educación Básica*. Santiago: Ediciones SM. Open access: http://www.smconecta.cl/refip/
- Sack, J. y Vazquez, I. (2016). A 3D visualization teaching-learning trajectory for elementary grades children. Países

Bajos: Springer. doi: 10.1007/978-3-319-29799-6

- Jurdak, M. (2016). *Learning and teaching real world problem solving in school mathematics*. The Netherland: Springer.
- Clements, D. H., Swaminathan, S., Zeitler, M. A. y Sarama, J. (1999). Young Children's Concepts of Shape. *Journal for Research in Mathematics Education*, *30*(2), 199-212.
- Howse, M. E. y Howse, T. D. (2015). Linking the Van Hiele theory to instruction. *Teaching Children Mathematics*, *21*(5), 304-313.

Bibliografía complementaria:

- Aldon, G. & Trgalova, J. (Eds.). (2019). *Teachnology in mathematics teaching*. The Netherlands: Springer.
- Andreasen, J. B. & Haciomeroglu, E. S. (2014). Engaging geometry students through technology. *Mathematics Teaching in the middle school*, *19*(5), 308-310.
- Chapin, S., O´Connor, & Anderson, N.C. (2005). Classroom discussions: Using math talk in elementary classrooms. Recuperado en http://mail.mathsolutions.com/documents/978- 1-935099-01-7_L.pdf
- Chick, L., Holmesn, A. S., McClymonds, N., Musick, S., Reynolds, P. y Schultz, G. (2008). Geometry and measurement. *Teaching Children Mathematics*, 14(7), 408-409.
- Gonzato, M., Godino, J. D. y Neto, T. (2011). Evaluación de conocimientos didáctico-matemáticos sobre la visualización de objetos tridimensionales. *Educación Matemática*, *23*, (3), 5-37.
- Hede, J. T., & Bostic, J. D. (2014). Connecting the Threads of Area and Perimeter. *Teaching Children Mathematics*, *20*(7), 418-425. https://doi.org/10.5951/teacchilmath.20.7.0418
- Herbst, P., Cheah, U. H., Richard, P. R., & Jones, K. (Eds.). (2018). *International perspectives on the teaching and learning of geometry in secondary school*. Países Bajos: Springer.
- International association for the evaluation of educational achievement (2009). Released ítems. Mathmatics – Forth Grade. Chestnut Hill, MA: IAE. Recuperado en: http://timssandpirls.bc.edu/timss2007/items.html
- Larraín, M. & Chandía. E. (2012). *La enseñanza de la geoemetría en la formación inicial de profesores de educación básica: una propuesta metodológica*. Santiago: Universidad del Desarrollo.
- Ministerio de Educación (2012). Bases Curriculares para la Educación Básica. Santiago, Chile: Unidad de Currículum y Evaluación.
- National Council of Teachers of Mathematics. (2011). Teaching geometry. *The Mathematics Teacher*, *105*(4), 244.
- Oldknow, A. & Knights, C. (2011). *Mathematics Education with digital Technology.* New York, NY: Continuum.
- Porkess, R. (Editor) (2014). Geometry and measures. Reino Unido: Hodder Education.
- Schettino, C. (2011). Teaching Geometry through Problem-Based Learning. The Mathematics Teacher, 105(5), 346-351. http://www.jstor.org/stable/10.5951/mathteacher.105.5.0346 .
- Usiskin, Z. (2010). *Future Curricular Trends in School Algebra And Geometry: Proceedings of A Conference*. Charlotte, NC: Information Age Publishing.

Informáticos:

https://www.nctm.org/sem/
https://www.nctm.org/Classroom-Resources/More-Online-Resources-from-NCTM/
https://www.nctm.org/pows/
https://www.nctm.org/crcc/
https://www.nctm.org/ARCs/
https://www.nctm.org/Standards-and-Positions/Focus-in-High-School-Mathematics/Reasoning-and-Sense-Making-Task-Library/

Softwares

https://www.didax.com/math/virtual-manipulatives.html
https://mathigon.org/polypad
https://phet.colorado.edu/en/simulations/filter?subjects=math&type=html&sort=alpha&view=grid
https://www2.ual.es/neotrie/

I. PRÁCTICAS DE ALTO IMPACTO (PAI)

Identifique con una X las PAI que serán trabajadas de forma principal y secundaria en el curso.

- PAI principales: son trabajadas y evaluadas de forma explícita y sistemática en el curso.

- PAI secundarias: son mencionadas y tocadas en el curso, pero no cumplen los criterios de las PAI principales.

Principal	Secundaria	Práctica de Alto Impacto
		1.Liderar discusiones grupales
X		2.Explicar y modelar los contenidos, prácticas y estrategias
		3.Elicitar e interpretar el pensamiento individual de los estudiantes
		4.Diagnosticar patrones comunes particulares en el razonamiento y desarrollo de los estudiantes en una asignatura
		5.Implementar normas y rutinas para el discurso y el trabajo de la sala de clases
		6.Coordinar y ajustar la enseñanza durante una clase
		7.Especificar y reforzar el comportamiento productivo de los estudiantes
		8.Implementar rutinas de organización
		9.Establecer y gestionar el trabajo de los estudiantes en grupos pequeños
		10.Construir relaciones respetuosas con los estudiantes
		11.Conversar acerca de los estudiantes con sus padres o apoderados
		12.Aprender sobre el contexto cultural, religioso, familiar, intelectual y personal de los estudiantes y considerarlo en el proceso de enseñanza - aprendizaje
		13. Establecer metas de corto y largo plazo para los estudiantes.

Principal	Secundaria	Práctica de Alto Impacto
		14.Planificar clases y secuencias de clases
		15.Comprobar la comprensión de los estudiantes durante y al final de cada clase
X		16.Seleccionar y diseñar evaluaciones formales del aprendizaje de los estudiantes
		17.Interpretar los resultados del trabajo de los estudiantes, incluyendo tareas cotidianas, controles, pruebas, proyectos y evaluaciones estandarizadas
		18.Retroalimentar a los estudiantes de forma oral y escrita
		19. Analizar la enseñanza con el propósito de mejorarla

Describa cómo se evaluarán las principales PAI identificadas, a lo largo del curso, incluyendo el examen final:

The PAI identified allow articulate the design of this course. In the first place, PAI ·Explaining and modeling the contents, practices and strategies· will be evaluated through the decisions you make when solving a problem that involves geometric or measurement thinking. These decisions, which have a direct connection with a high disciplinary and didactic domain —both in Spanish and in English— should account for the use of different strategies, representations and forms to deal with mathematical concepts, in order to use mathematical knowledge. meaningfully, effectively and profoundly. In conclusion, each of the decisions you make (which can be embodied in the different evaluation strategies or in training activities) must account for the importance of paying attention to the relationships that exist between the geometric/measurement contents.

Second, the PAI “selecting and designing formal evaluations of student learning” will be evaluated through the recognition of different needs in which it makes sense to collect evidence on the mathematical learning of students and, based on said results, take decisions. Specifically, we will evaluate that you respond to a specific need (a common mistake, for example) and design strategies or instruments to evaluate the issues they are dealing with.

J. PROFESIONALISMO DOCENTE (prácticas éticas)

Identifique con una X el o los comportamientos profesionales que serán enseñados y evaluados de forma explícita y sistemática en el curso.

	Comportamiento
x	1. Respeta el carácter único de cada estudiante y, por tanto, la diversidad que se manifiesta entre ellos.
x	2. Se hace responsable del acceso equitativo al aprendizaje y del desarrollo del máximo potencial de todos los estudiantes.
	3. Actúa con honestidad e integridad.
	4. Demuestra un trabajo riguroso y responsable.
	5. Ejerce cuidadosamente el liderazgo y la autoridad que implica el rol docente.
	6. Trabaja de manera colaborativa y respetuosa con jefaturas, colegas, padres y apoderados, técnicos y otros miembros de la comunidad educativa.
	7.Mejora continuamente su desempeño profesional.

Describa cómo se evaluará el profesionalismo docente a lo largo del curso, incluyendo el examen final:

The way this course addresses geometry/measurement issues reflects consideration of the diversity of students present in the classroom in general, and the diversity of thinking about mathematics, in particular. We will guide you towards the discovery of multiple paths to communicate a mathematical concept or procedure, in order for you to make timely decisions that are justified by the unique character of each student. In the same way, we adhere to the idea that each proposal or decision you make about ways to guide and build your students' learning promotes that everyone has access to learning in a timely, meaningful and clear manner.